A new window on primordial non-Gaussianity

based on
1201.5375 with M. Zaldarriaga

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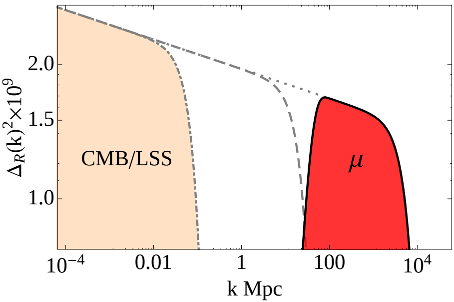
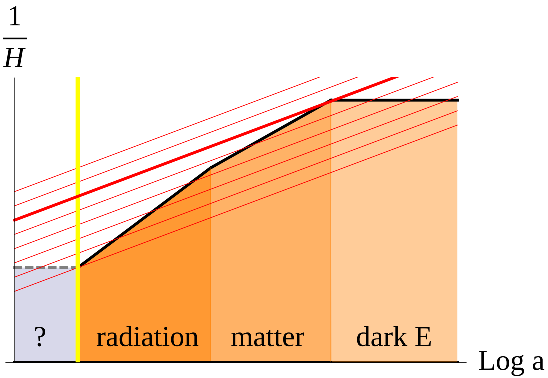


Summary

- We know little about primordial perturbations outside the range $10^{-4} \lesssim k\text{Mpc} \lesssim 1$
- μ-type spectral distortion of the CMB is a unique probe of small scales $50 \lesssim k\text{Mpc} \lesssim 10^4$

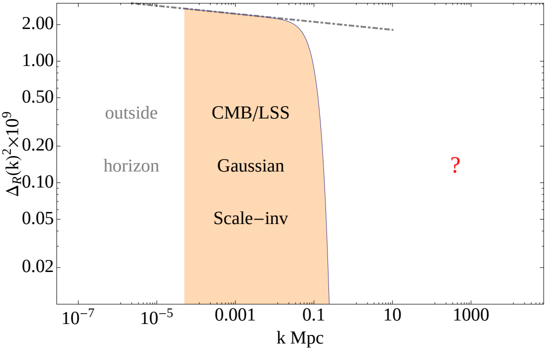
  [Sunyaev, Zel'dovich, Silk, Peebles, Hu, Danese, de Zotti, Chluba, ...]
- The monopole $\langle \mu(\hat{n}) \rangle$ probes the small-scale power spectrum
- μT cross correlation probes the primordial bispectrum in the squeezed limit f_{NL}^{loc}
- Fisher forecast with current technology $\Delta f_{NL}^{loc} \lesssim 10^3$
- Beat cosmic variance with an enormous number of modes

Primordial perturbations



- Primordial superhorizon perturbations seed the structures in our universe
- They teach us about the earlier stage

Primordial perturbations: What do we know?



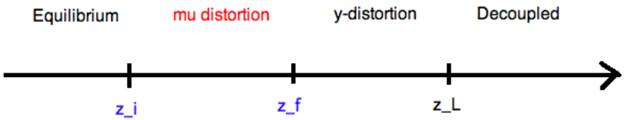
- $k \lesssim 10^{-4}\,\mathrm{Mpc}^{-1}$ are still outside our horizon
- $k \gtrsim 0.15\,\mathrm{Mpc}^{-1}$ ($l \gtrsim 2000$) have been erased by Silk damping
- $k \gtrsim \mathcal{O}(1)\,\mathrm{Mpc}^{-1}$ are now contaminated by gravitational non-linearities

Photon thermodynamics before decoupling

- Before $z_i \simeq 2 \times 10^6$ double Compton scattering ($e^- + \gamma \to e^- + 2\gamma$) is very efficient. Perfect thermodynamical equilibrium, Planck spectrum $n(\nu) = (e^{\nu/k_B T} - 1)^{-1}$
- Between z_i and $z_f \sim 5 \times 10^4$ only elastic Compton scattering ($e^- + \gamma \to e^- + \gamma$) is efficient. Photon number is effectively frozen. Bose-Einstein spectrum with chemical potential μ

$$n(\nu) = \frac{1}{e^{\nu/k_B T + \mu} - 1}$$

- After z_f also elastic Compton scattering is not efficient, e.g. y-type distortion.

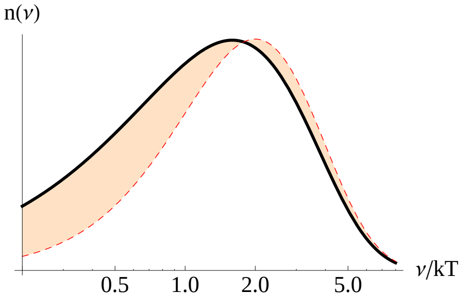


μ-distorted spectrum

For $\mu > 0$ the spectrum

$$n(\nu) = \frac{1}{e^{\nu/k_B T + \mu} - 1}$$

looks like



The distortion has a different ν dependence from y-distortion.

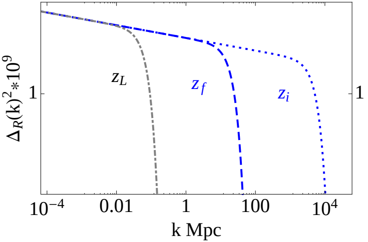
μ-distortion probes $50 \lesssim k\mathrm{Mpc} \lesssim 10^4$

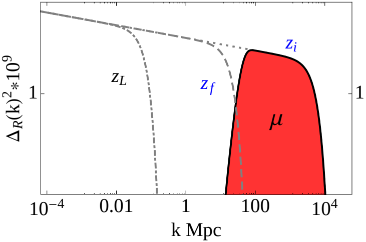
- Perturbations of the adiabatic mode $\mathcal{R}$ re-enter the horizon and oscillate and dissipate

$$\delta_\gamma \sim \mathcal{R}_k \cos(kt)\, e^{-k^2/k_D^2}$$

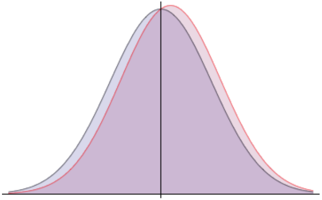
- Damping of $k < k_D \sim z^{3/2}$ erases primordial perturbations and injects δE into photons
- For $z_i < z < z_f$ μ-distortion is created $1.4\delta E = \mu \sim \mathcal{R}^2$

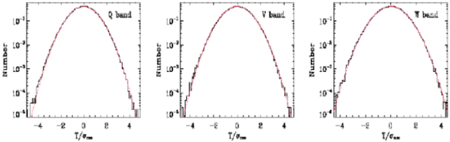
Out[90]=





It is hard to tell by eye





- For a Gaussian random variable

$$\langle \delta^{2n+1} \rangle = 0 \,, \quad \langle \delta^{2n} \rangle \propto \langle \delta^2 \rangle^n$$

- Non-vanishing odd correlation $\rightarrow$ non-Gaussianity
- $\delta \ll 1 \rightarrow \langle \delta^3 \rangle$ is the most sensitive

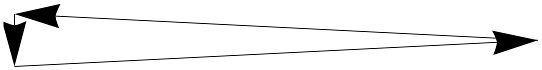
Symmetries, sizes and shapes

- Conservation of momentum + rotational invariance + scale invariance

$$\langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3)\rangle \equiv (2\pi)^3 f_{NL} F\left(k_1, k_2, k_3\right) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

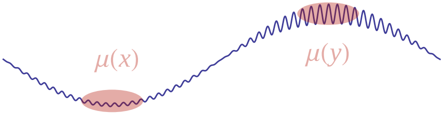
- Interesting limit $k_3 \ll k_1 \sim k_2$
- f_{NL}^{loc} distinguishes between single and multifield inflation

$$F^{\text{loc}} \sim \frac{\Delta_{\mathcal{R}}^2}{k_1^3} \frac{\Delta_{\mathcal{R}}^2}{k_2^3} + 2\text{perm}'\text{s}$$



μT cross correlation

- Spherical harmonics: $\mu(\hat{n}),\, T(\hat{n}) \rightarrow a^{\mu}_{lm},\, a^{T}_{lm}$
- μT gives the primordial bispectrum in the very squeezed limit f^{loc}_{NL}



- Straightforward computation

$$\langle a^{\mu}_{lm} a^{T}_{lm} \rangle \equiv C^{\mu T}_{l} \quad \simeq \quad 50\, \frac{\Delta^4_{\mathcal{R}}(k_p)}{l(l+1)}\, f^{loc}_{NL}\, b \simeq \frac{3 \times 10^{-16}}{l(l+1)}\, f^{loc}_{NL}\, b$$

- $b \sim \Delta^2_{\mathcal{R}}(k_D)/\Delta^2_{\mathcal{R}}(k_p)$, if scale invariant $b \sim 1$.

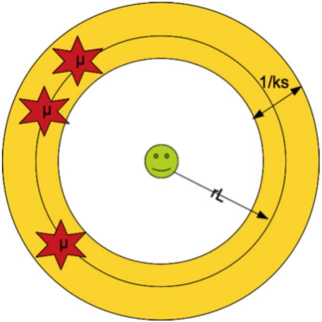
$\mu\mu$ Gaussian self correlation

- $\mu\mu$ receives both a Gaussian and a non-Gaussian contributions. The Gaussian is

$$\langle a^{\mu}_{lm} a^{\mu}_{lm} \rangle \equiv C^{\mu\mu}_{l,\mathrm{Gauss}} \sim 6 \times 10^{-17} \frac{\Delta^4_{\mathcal{R}}(k_{D,f})}{\Delta^4_{\mathcal{R}}(k_p)} \frac{k_s r_L^{-2}}{k^3_{D,f}} \lesssim 1.5 \times 10^{-28}$$

- White noise, l-independent
- Very small cosmic variance! Suppressed by $N_{\mathrm{modes}}^{-1/2}$ with

$$N_{\mathrm{modes}} \sim \frac{k^3_{D,f}}{k_s r_L^{-2}} \sim 10^{12}$$



Fisher matrix forecast

- Signal to noise for f_{NL}^{loc} from $C_l^{\mu T}$

$$\left(\frac{S}{N}\right)^2 = \sum_l \frac{C_l^{\mu T} C_l^{\mu T}}{\frac{1}{2l+1} C_l^{TT} C_l^{\mu\mu,N}}$$

- A figure of merit PIXIE [Chuss et al. '11]

$$\frac{S}{N} \simeq 10^{-3}\, b\, f_{NL}^{loc} \left(\frac{\sqrt{4\pi} \times 10^{-8}}{w_\mu^{-1/2}}\right) \sqrt{\log \frac{l_{\max}}{80}}.$$

i.e. $\Delta f_{NL}^{loc} \lesssim 10^3$ with current technology

How well can we do?

- Nature puts a lower bound on the noise, i.e. cosmic variance
- We can beat it only having more modes by $N_{\rm modes}^{-1/2}$
- For the TTT bispectrum

$$\frac{S}{N} \propto N_{\rm modes}^{1/2} \sim l_{\rm max} \log^{1/2}(l_{\rm max})$$

  Diffusion damping $\Rightarrow l_{\rm max} \lesssim 2000$. Ideal experiment $\Delta f_{NL}^{loc} \lesssim 3$

- For μT there are many more modes. Nature beats down cosmic variance for us

$$\frac{S}{N} \propto N_{\rm modes}^{1/2} \sim \sqrt{\frac{k_{D,f}^3}{k_s r_L^{-2}}} \sim 10^6$$

  Ideal experiment $\Delta f_{NL}^{loc} \lesssim 10^{-3}$

Conclusions

- μ-distortion probes small, other wise unaccesible scales
- μT is a direct and clean probe of the primordial bispectrum in the squeezed limit, f_{NL}^{loc}
- Cosmic variance is very small, allowing in principle for a large margin of improvement
- How would a dedicated experiment look and perform? Foregrounds?
- Numerical analysis is needed for detail predictions