A new window on primordial non-Gaussianity

based on
1201.5375 with M. Zaldarriaga

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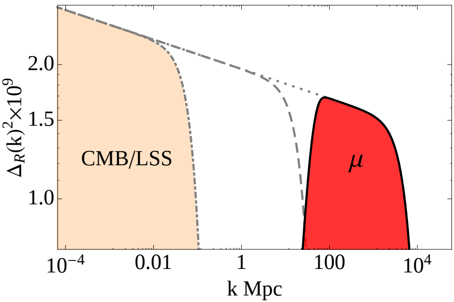
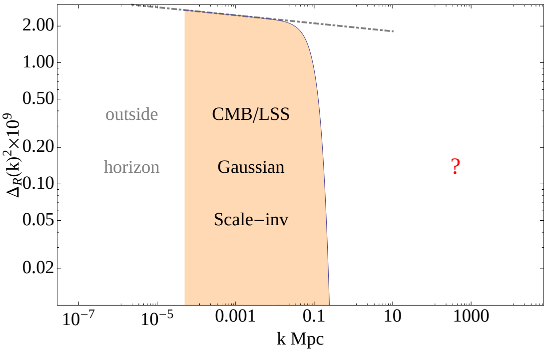


Summary

- We know little about primordial perturbations outside the range $10^{-4} \lesssim k\mathrm{Mpc} \lesssim 1$
- μ-type spectral distortion of the CMB is a unique probe of small scales $50 \lesssim k\mathrm{Mpc} \lesssim 10^4$

  [Sunyaev, Zel'dovich, Silk, Hu, Danese, de Zotti, Chluba, ...]
- The monopole $\langle \mu(\hat{n}) \rangle$ probes the small-scale power spectrum
- μ-T cross correlation probes the primordial bispectrum in the squeezed limit f_{NL}^{loc}
- Fisher forecast with current technology $\Delta f_{NL}^{loc} \lesssim 10^3$
- Beat cosmic variance with an enormous number of modes

Primordial perturbations: What do we know?



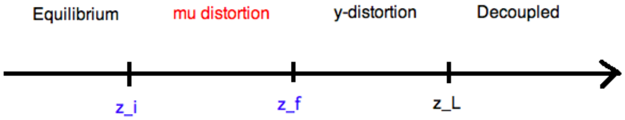
- $k \lesssim 10^{-4}\,\mathrm{Mpc}^{-1}$ are still outside our horizon
- $k \gtrsim 0.15\,\mathrm{Mpc}^{-1}$ ($l \gtrsim 2000$) have been erased by Silk damping
- $k \gtrsim \mathcal{O}(1)\,\mathrm{Mpc}^{-1}$ are now contaminated by gravitational non-linearities

Photon thermodynamics before decoupling

- Before $z_{\mu,i} \simeq 2 \times 10^6$ double Compton scattering $(e^- + \gamma \to e^- + 2\gamma)$ is very efficient. Perfect thermodynamical equilibrium, Planck spectrum $n(\nu) = (e^{\nu/k_B T} - 1)^{-1}$
- Between $z_{\mu,i}$ and $z_{\mu,f} \sim 5 \times 10^4$ only elastic Compton scattering $(e^- + \gamma \to e^- + \gamma)$ is efficient. Photon number is effectively frozen. Bose-Einstein spectrum with chemical potential μ

$$n(\nu) = \frac{1}{e^{\nu/k_B T + \mu} - 1}$$

- After $z_{\mu,f}$ also elastic Compton scattering is not efficient, e.g. y-type distortion.

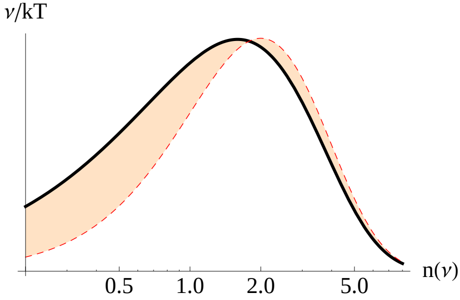


μ-distorted spectrum

For $\mu > 0$ the spectrum

$$n(\nu) = \frac{1}{e^{\nu/k_B T + \mu} - 1}$$

looks like



The distortion has a different ν dependence from y-distortion.

μ-distortion probes $50 \lesssim k\text{Mpc} \lesssim 10^4$

- Perturbations of the adiabatic mode $\mathcal{R}$ re-enter the horizon and oscillate and dissipate

$$\delta_\gamma \sim \mathcal{R}_k \cos(kt)\, e^{-k^2/k_D^2}$$

- Damping of $k < k_D$ erases primordial perturbations and injects δE into photons
- For $z_{\mu,i} < z < z_{\mu,f}$ μ-distortion is created $\mu = 1.4\delta E$

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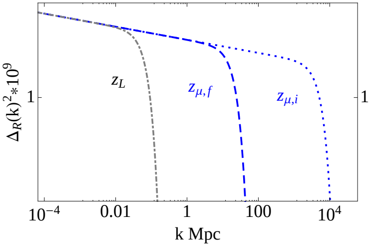
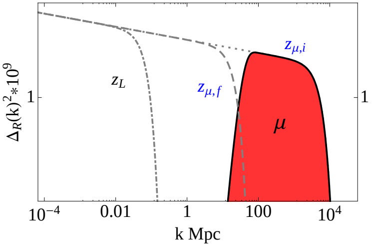




μ measures $\mathcal{R}^2$ at small scales

- Energy of acoustic wave $\rho c_s^2 \langle \delta_\gamma(x)^2 \rangle_p$. Dissipated energy:

$$\delta\mu \sim \delta E \sim \frac{\rho}{3} \langle \delta_\gamma(x)^2 \rangle_p |_f^i$$

- For adiabatic perturbation

$$\delta_\gamma \sim \mathcal{R}_k \cos(kt)\, e^{-k^2/k_D^2}$$

with $k_D \propto z^{3/2}$

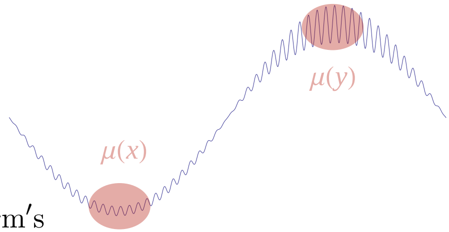
- Smearing over $1/k_s$

$$\mu(x) \simeq 6 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \mathcal{R}(\vec{k}_1)\mathcal{R}(\vec{k}_2) e^{i\vec{k}_+\cdot\vec{x}} W\left(\frac{k_+}{k_s}\right) \times \langle \cos(k_1 t) \cos(k_2 t) \rangle_p \left[e^{-(k_1^2+k_2^2)/k_D^2} \right]_f^i$$

μ-T cross correlation

- Spherical harmonics: $\mu(\hat{n}), T(\hat{n}) \rightarrow a^{\mu}_{lm}, a^{T}_{lm}$
- μ-T gives the primordial bispectrum in the very squeezed limit f_{NL}^{loc}

$$\langle \mathcal{R}^3 \rangle \sim f_{NL}^{loc} \Delta^2_{\mathcal{R}}(k_1) \Delta^2_{\mathcal{R}}(k_2) + \text{perm}'\text{s}$$



- Straightforward computation

$$C_l^{\mu T} \simeq 50 \frac{\Delta^4_{\mathcal{R}}(k_p)}{l(l+1)} f_{NL}^{loc}\, b \simeq \frac{3 \times 10^{-16}}{l(l+1)} f_{NL}^{loc}\, b$$

- $b \sim \Delta^2_{\mathcal{R}}(k_D)/\Delta^2_{\mathcal{R}}(k_p)$, if scale invariant $b \sim 1$.

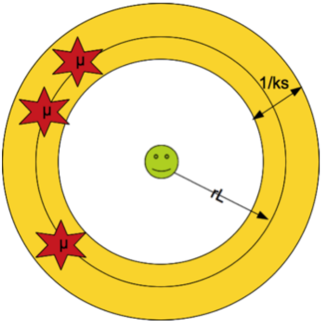
μ-μ Gaussian self correlation

- μ-μ receives both a Gaussian and a non-Gaussian contributions. The Gaussian is

$$C_{l,\text{Gauss}}^{\mu\mu} \sim 6 \times 10^{-17} \frac{\Delta_{\mathcal{R}}^4(k_{D,f})}{\Delta_{\mathcal{R}}^4(k_p)} \frac{k_s r_L^{-2}}{k_{D,f}^3}$$
$$\lesssim 1.5 \times 10^{-28}$$

- μ fluctuations are uncorrelated at distances $\Delta x \gg 1/k_s$
- White noise, l-independent
- Very small cosmic variance! Suppressed by $N_{\text{modes}}^{-1/2}$ with

$$N_{\text{modes}} \sim \frac{k_{D,f}^3}{k_s r_L^{-2}} \sim 10^{12}$$



μ-μ non-Gaussian self correlation

- The non-Gaussian contribution to μ-μ probes the trispectrum

$$\langle \mathcal{R}^4 \rangle \sim \tau_{NL} \Delta_R^2(k_1) \Delta_R^2(k_2) \Delta_R^2(k_{13}) + \mathrm{perm's}$$

- One finds

$$C_{l,NG}^{\mu\mu} \quad \sim \quad 9 \times 10^{-23} \, \tau_{NL} \, \frac{b'}{l(l+1)} \, ,$$

- $b' \sim \Delta_{\mathcal{R}}^4(k_D)/\Delta_{\mathcal{R}}^4(k_p)$, if scale invariant $b' \sim 1$.
- $C_l^{\mu\mu}$ is more sensitive to non-Gaussianity than C_l^{TT}, since there is less cosmic variance.
- For small non-Guassianity there is more signal in $C_l^{\mu T}$

Fisher matrix forecast

- Signal to noise for f_{NL}^{loc} from $C_l^{\mu T}$

$$\left(\frac{S}{N}\right)^2 = \sum_l \frac{C_l^{\mu T} C_l^{\mu T}}{\frac{1}{2l+1} C_l^{TT} C_l^{\mu\mu,N}}$$

- Simple model for the noise

$$C_l^{\mu\mu,N} \simeq w_\mu^{-1}\, e^{l^2/l_{\max}^2}\,,$$

- A figure of merit PIXIE [Chuss et al. '11]

$$\frac{S}{N} \simeq 10^{-3}\, b\, f_{NL}^{loc} \left(\frac{\sqrt{4\pi}\times 10^{-8}}{w_\mu^{-1/2}}\right) \sqrt{\log \frac{l_{\max}}{80}}.$$

i.e. $\Delta f_{NL}^{loc} \lesssim 10^3$ with current technology

How well can we do?

- Nature puts a lower bound on the noise, i.e. cosmic variance
- We can beat it only by $N_{\rm modes}^{-1/2}$, i.e. having more modes
- For the TTT bispectrum

$$\frac{S}{N} \propto N_{\rm modes}^{1/2} \sim l_{\rm max} \log^{1/2}(l_{\rm max})$$

Because of Silk damping one can not do better than $l_{\rm max} \sim 2000$.
Ideal experiment $\Delta f_{NL}^{loc} \lesssim 3$

- For μT there are many more modes. Nature beats down cosmic variance for us

$$\frac{S}{N} \propto N_{\rm modes}^{1/2} \sim \sqrt{\frac{k_{D,f}^3}{k_s r_L^{-2}}} \sim 10^6$$

Ideal experiment $\Delta f_{NL}^{loc} \lesssim 10^{-3}$

Addressing some questions

From Fabian's introduction to the workshop

- Is there something to look for on smaller scales (than reachable by CMB C(l)) ?

Yes! E.g. μ-distortion probes $50 \lesssim k\text{Mpc} \lesssim 10^4$

- Are ongoing efforts in CMB/LSS sufficient in elucidating inflationary physics ? Optimal ? Do we need more focused efforts ?

Experiments dedicated to spectral distortion can teach us a lot about the early universe

Conclusions

- μ-distortion probes small, other wise unaccesible scales
- μT is a direct and clean probe of the primordial bispectrum in the squeezed limit, f_{NL}^{loc}
- Cosmic variance is very small, allowing in principle for a large margin of improvement
- How would a dedicated experiment look and perform? Foregrounds?
- Numerical analysis is needed for detail predictions

Conclusions

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$\langle\mu(\hat{n})\rangle$ constraints the small-scale power spectrum

- The averaged μ-distortion over the whole sky (monopole) [Hu et al. '94]

$$\langle\mu(x)\rangle \simeq 3\int d\log k\, \Delta^2_{\mathcal{R}}(k)\left[e^{-2k^2/k_D^2}\right]_f^i ,$$

- Integral measure of the power spectrum at small scales
- For scale invariant $\mu \sim 5\times10^{-8}$, but there could be surprises!
- Needs an absolute measurement of the CMB spectrum